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PHYSICS LETTERS B

Physics Letters B 551 (2003) 111–114

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Two-loop bosonic electroweak corrections to the muon lifetime and M_Z–M_W interdependence

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Received 16 September 2002; received in revised form 7 November 2002; accepted 11 November 2002

Editor: P.V. Landshoff

Abstract

The two-loop electroweak bosonic correction to the muon lifetime and the M_W–M_Z interdependence is computed using analytical methods of asymptotic expansion. Combined with previous calculations this completes the full two-loop correction to Δr in the Standard Model. The shift to the prediction of W-boson mass due to two-loop bosonic corrections does not exceed 1 MeV for the range of the Higgs boson mass from 100 to 1000 GeV.

The Fermi constant G_F plays an important role in the precision tests of the Standard Model. Theoretically G_F can be related to other precision observables: the electroweak coupling constant α and the masses of electroweak gauge bosons M_Z and M_W. Other parameters enter this expression through quantum corrections. Usually one inverts this relation in order to predict M_W through M_Z which is measured much more accurate. This M_Z–M_W interdependence can be then confronted with experimental value $M_W^{\exp}$. The current error (39 MeV) of $M_W^{\exp}$ will be drastically reduced at future colliders. In fact, at LHC the experimental error can be reduced to 15 MeV [1] and at Linear Collider even down to 6 MeV [2]. Therefore, much efforts have been spent to reduce the error of the theoretical prediction.

Fermi constant is defined as the coupling constant in the low energy four fermion effective Lagrangian describing the decay of the muon

$$\mathcal{L}_F = \mathcal{L}_{QED} + \frac{G_F}{\sqrt{2}}\left[\bar{\nu}_\mu\gamma^\alpha(1-\gamma_5)\mu\right]\left[\bar{e}\gamma_\alpha(1-\gamma_5)\nu_e\right], \quad (1)$$

where e and μ are electron and muon fields, ν_e and ν_μ are the corresponding neutrinos and G_F is the Fermi constant. From the Lagrangian (1) one gets the following value for the muon lifetime

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_e^2}{m_\mu^2}\right)(1+\Delta q), \quad (2)$$

where the factor Δq describes all the quantum corrections in the low energy effective theory (i.e., QED corrections). At one-loop order these corrections have been computed a long time ago [3]. Recently also the

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[1] Supported by DFG-Forschergruppe *"Quantenfeldtheorie, Computeralgebra und Monte-Carlo-Simulation"* (contract FOR 264/2-1).

[2] Supported by BMBF under grant No. 05HT9VKB0.

0370-2693/02
PII: S0370-2693(02)03004-6

two-loop result for Δq has been obtained [4]. Taking it into account, the error of G_F is nowadays dominated by the experimental error of τ_μ measurement.[3]

In order to relate G_F to M_W one can use the matching condition between effective theory (1) and the Standard Model, which requires that the value of τ_μ does not depend on whether it is evaluated in the Fermi theory or in the full Standard Model. This brings us to the relation

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8 s_W^2 M_W^2}(1+\Delta r), \tag{3}$$

where e is the electric charge, $s_W = \sin\theta_W$ is the SM mixing angle and M_W is the W-boson mass. The factor $e^2/8s_W^2 m_W^2$ corresponds to the matching at the three level, while the quantity $\Delta r = \Delta r^{\text{1-loop}} + \Delta r^{\text{2-loop}} + \cdots$ is defined to absorb quantum corrections coming from the matching procedure at higher loop orders. At the one-loop level Δr was first evaluated in [5]. Further improvement of Δr was achieved in [6] where the leading and subleading large t-quark corrections were computed at $O(\alpha^2)$ order. Other two-loop fermionic corrections were considered in a series of papers [7].

The aim of this Letter is to compute the remaining $O(\alpha^2)$ pure bosonic contribution in order to complete the full $O(\alpha^2)$ approximation to Δr. As this work was in progress Ref. [8] has appeared where this missing contribution has been computed numerically using the integral representation for the massive two-point functions [9].

Our approach is complementary to that of [8]. We use the methods of asymptotic expansion [10] in two different regimes: (1) large Higgs mass expansion and (2) expansion in difference of masses $M_H^2 - M_Z^2$. In addition we consider also $s_W^2 = \sin^2\theta_W$ as a small parameter in the spirit of [11].

The details of our calculation will be presented in a separate publication [12] and here we only sketch the most important moments.

- G_F is nothing but the Wilson coefficient function of the corresponding operator in the low energy four fermion effective theory. Therefore, it depends only on the "hard" physics and is insensitive to low energy field modes in loops. The simplest way to compute such a quantity is the use of the factorization theorem (if it exists), which allows one to separate "hard" and "soft" physics.[4] This theorem for the construction of low energy effective theories is known for a long time (see, e.g., [13]) and is based on the Euclidean structure of large mass expansion.
- Using the factorization theorem the problem is reduced to the evaluation of two-loop bubble diagrams.
- All the calculations have been performed in a general R_ξ gauge with three arbitrary gauge parameters. By explicit calculation we checked that contribution to Δr is gauge invariant.
- In order to have explicit gauge invariant expressions also at the level of bare quantities we explicitely include tadpole diagrams. We have checked that applying this prescription all the bare parameters and all the counterterms for the masses and the coupling constant are explicitely gauge invariant.
- The renormalization was performed in two different schemes: $\overline{\text{MS}}$ and on-shell scheme. For the transformation from one scheme to another one needs the results of [11] where the transformation formulae for the gauge boson masses were given to two loops.

To obtain the two-loop bosonic contribution to Δr a number of two-loop diagrams of the order of 10^4 should be evaluated. In our calculation the computer algebra system FORM [14] was used. In order to obtain the FORM readable input we make use of the input generator DIANA [15].

Let us now present the results of the calculation. In Fig. 1 the results of expansions are plotted as a function of the on-shell Higgs boson mass M_H starting from the experimental low bound 114 GeV [16] up to 1 TeV. First we expand in the limit $M_H \to M_Z$, e.g., in the difference $(M_H^2 - M_Z^2)$. By summing this series we have found that with 6–10 coefficients it works

[3] The present value is $G_F = 1.16637(1) \times 10^{-5}\ \text{GeV}^{-2}$ and possible future experiments could reduce the error by an order of magnitude.

[4] In our case by "soft" we understand energies and momenta of order of muon mass m_μ, while "hard" means scales $\gg m_\mu$.

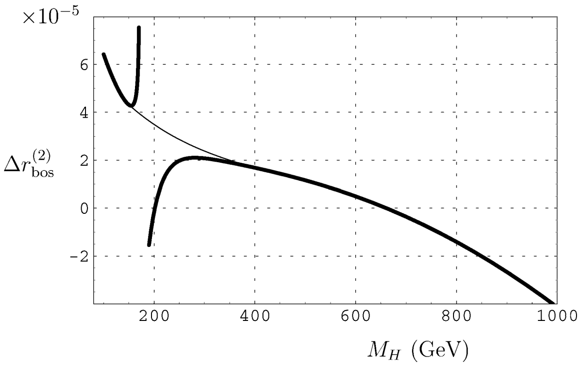

Fig. 1. Dependence of $\Delta r^{(2)}_{\rm bos}$ on Higgs mass M_H (see explanation in the text). Parameter choice for the pole masses: $M_Z = 91.1876$ GeV and $M_W = 80.423$ GeV.

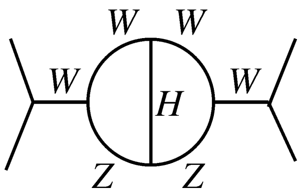

Fig. 2. Example of a diagram that has singularity near $M_H = 2M_W$.

well for the values of M_H up to ~ 125 GeV. In order to accelerate the convergence of this series we apply Páde approximation w.r.t. the variable $y = (M_H^2 - M_Z^2)/M_Z^2$. The left bold curve in Fig. 1 represents the [3/3] Páde approximant which behaves smooth up to ~ 170 GeV. The other bold curve is a sum of the first 4 terms of large Higgs mass expansion. This series appeares to be non-alternating and the application of Páde did not show any considerable improvement. In the intermediate region 155 GeV $< M_H <$ 350 GeV both approximations fail to converge. (This is due to the presence of singularities in M_H-plane at $|M_H| = 2M_W$ or $|M_H| = 2M_Z$. Such singularity appears, for example, in diagram of Fig. 2. This manifests itself as a divergence of the series as M_H approaches $2M_Z$.) Therefore, in order to get a result for Higgs mass values in the region 155–350 GeV we made simple polynomial interpolation.

Our comparison with the numerical curve from Ref. [8] shows a perfect agreement between the two calculations. The difference for 155 GeV $< M_H <$ 350 GeV does not exceed 5% while for M_H below 155 GeV and above 350 GeV it is much smaller.

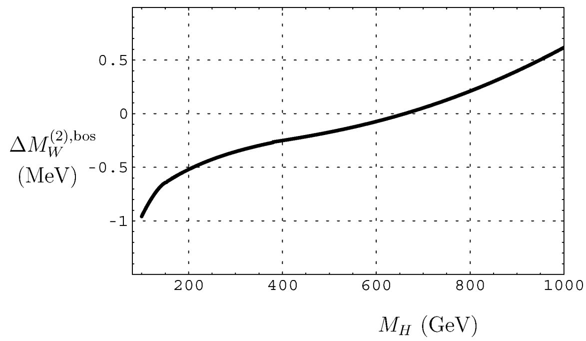

Fig. 3. Shift ΔM_W of the predicted mass of W-boson due to two-loop bosonic correction to Δr.

Let us now turn to the M_Z–M_W interdependence. By inverting Eq. (3) we can find how much does the computed $\Delta r^{(2)}_{\rm bos}$ contribute to the shift of the value of the W-boson mass. This contribution is plotted in Fig. 3. For the broad region of M_H from 100 to 1000 GeV this correction does not exceed 1 MeV and appears to be insignificant. Therefore, we can see that the uncertainty in M_W prediction at the moment is dominated by the uncertainty in the experimental value of t-quark pole mass.

In conclusion, the bosonic contributions to Δr has been computed using methods of asymptotic expansions and the low energy factorization theorem. The influence of this correction on the theoretical prediction of M_W is found. The gauge invariance of $\Delta r^{(2)}_{\rm bos}$ has been proved explicitly. By the inclusion of Higgs-tadpole diagrams it has been checked that all mass- and charge-counterterms are gauge invariant.

Acknowledgements

Authors would like to thank M. Kalmykov, M. Awramik, M. Czakon, K. Chetyrkin and J. Kühn for fruitful discussions and F. Campanario for the careful reading the Letter. We are also grateful to M. Tentyukov for his help with input generator DIANA.

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